Quiz # 4 Solutions

1.

$$f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}.$$

2. (a)

$$\begin{aligned} f'(2) &= \lim_{h\to 0} \frac{[3(2+h)^2 - 5(2+h)] - [3\cdot 2^2 - 5\cdot 2]}{h} \\ &= \lim_{h\to 0} \frac{3(4+4h+h^2) - 10 - 5h - (12-10)}{h} \\ &= \lim_{h\to 0} \frac{12 + 12h + 3h^2 - 5h - 12}{h} = \lim_{h\to 0} \frac{h\,(12+3h-5)}{h} = \lim_{h\to 0} 12 + 3h - 5 = 7 \end{aligned}$$

(b)

$$y - 2 = 7(x-2)$$

(all the numbers "y_0," "x_0," and "m" are available already!)

3.

$$\begin{aligned} g'(x) &= \lim_{h\to 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h} = \lim_{h\to 0} \frac{\frac{x^2-(x+h)^2}{(x+h)^2x^2}}{h} = \lim_{h\to 0} \frac{x^2 - (x^2+2xh+h^2)}{h(x+h)^2x^2} \\ &= \lim_{h\to 0} \frac{-2xh - h^2}{h(x+h)^2x^2} = \lim_{h\to 0} \frac{h(-2x-h)}{h(x+h)^2x^2} = \lim_{h\to 0} \frac{-2x-h}{(x+h)^2x^2} = \frac{-2x-0}{(x+0)^2x^2} = \frac{-2x}{x^4} \\ &= \frac{-2}{x^3} \end{aligned}$$

4. [*Sketch will be given in class.*]