Quiz # 3 Solutions

1. I rewrite "$\cos x = x$" as the equivalent equation $\cos x - x = 0$. The function $f(x) = \cos x - x$ is continuous. And

$$f(0) = \cos 0 - 0 = 1 \qquad \text{while} \qquad f(1) = \cos 1 - 1.$$

Now, I don't know what $\cos 1$ is, but it is not 1 and it is less than 1. (The only inputs for which $\cos x$ gives value one are the even multiples of π, and 1 is not an even multiple of π.) Thus $\cos 1 - 1 < 0$. So

$$f(0) > 0 \qquad \text{while} \qquad f(1) < 0$$

and f is continuous on $(0, 1)$. The Intermediate Value Theorem shows there is some x for which $f(x) = 0$, that is, there is a solution to the equation $\cos x = x$ on $(0, 1)$.

(*It turns out that* $x = 0.739085133215$ *or so, but that is irrelevant to the question.*)

2. (a) For $x \neq 1$ the given function simplifies:

$$f(x) = \frac{x^2 - x}{x^2 - 1} = \frac{x(x-1)}{(x-1)(x+1)} = \frac{x}{x+1}.$$

(*I emphasize that this calculation is correct for* x not equal to 1.) Thus

$$\lim_{x \to 1} f(x) = \lim_{x \to 1} \frac{x}{x+1} = \frac{1}{2}.$$

Unfortunately, $f(1) = 1$ as stated in the problem. Thus

$$\lim_{x \to a} f(x) \neq f(a)$$

when a is 1. Thus f is not continuous at $a = 1$.

(b) [*Sketch will be given in class.*]

3. (a) $\lim_{x \to \infty} \cos x$ does not exist (*because there is no value* L *which the values* $\cos x$ *get close to, and stay close to, when* x *is large*)

(b)

$$\lim_{x \to \infty} \frac{3x + 5}{x - 4} = \lim_{x \to \infty} \frac{3x + 5}{x - 4} \frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x \to \infty} \frac{3 + \frac{5}{x}}{1 - \frac{4}{x}} = \frac{3 + 0}{1 - 0} = 3$$

4. [*Sketch will be given in class.*]