Quiz # 11 Solutions

1. (a)

$$\int 10x^4 + \csc^2 x\,dx = \frac{10}{5}x^5 - \cot x + C = 2x^5 - \cot x + C.$$

(b) Use the substitution $u = x^3 + 5$ so $du/3 = x^2 dx$:

$$\int x^2 \left(x^3+5\right)^9\,dx = \int (u)^9 \frac{du}{3} = \frac{1}{3}\int u^9\,du = \frac{1}{3\cdot 10}u^{10} + C = \frac{1}{30}(x^3+5)^{10} + C$$

2.

$$\begin{aligned}\int \tan x\,dx &= \int \frac{\sin x}{\cos x}\,dx = \int \frac{1}{u}\,(-du) \\ &= -\ln|u| + C = -\ln|\cos x| + C = \ln\left(|\cos x|^{-1}\right) + C = \ln|\sec x| + C\end{aligned}$$

3. Using the substitution $w = \sin u$,

$$\int_0^{\pi/2} \cos u\,\sin(\sin u)\,du = \int_0^1 \sin w\,dw = -\cos w\Big]_{w=0}^{w=1} = -\cos 1 + \cos 0 = 1 - \cos 1.$$

4. By the Fundamental Theorem of Calculus (II),

$$\int_0^5 v(t)\,dt = \int_0^5 s'(t)\,dt = s(5) - s(0) = 3 - 0 = 3.$$