Quiz # 10 Solutions

1. The definition is

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\,\Delta x$$

where $\Delta x = (b-a)/n$, $x_i = a + i\Delta x$, and x_i^* is some sample point in the interval $[x_{i-1}, x_i]$.

2. First, $\Delta x = 1/n$ and since we are using right end points,

$$x_i^* = x_i = 0 + i\Delta x = \frac{i}{n}.$$

The calculation then goes like this

$$\begin{aligned}\int_0^1 x^3\,dx &= \lim_{n\to\infty} \sum_{i=1}^{n} (x_i)^3\,\Delta x = \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{i}{n}\right)^3 \frac{1}{n} \\ &= \lim_{n\to\infty} \frac{1}{n^4} \sum_{i=1}^{n} i^3 = \lim_{n\to\infty} \frac{1}{n^4} \left[\frac{n(n+1)}{2}\right]^2 \\ &= \lim_{n\to\infty} \frac{n^2(n+1)^2}{4n^4} = \frac{1}{4} \lim_{n\to\infty} \frac{(n+1)^2}{n^2} = \frac{1}{4}.\end{aligned}$$

3. Here the calculation is quick because we know an antiderivative for x^3:

$$\int_0^1 x^3\,dx \overset{\text{FTC II}}{=} \frac{1}{4}x^4\Big]_0^1 = \frac{1}{4} - 0 = \frac{1}{4}.$$

Needless to say the answers must come out the same!

4. By the FTC I,

$$\frac{d}{dx}\left(\int_7^x e^t \sin(t^2)\,dt\right) = e^x \sin(x^2).$$

That is, differentiation and integration undo each other (i.e. are inverse operations).