

If you've been reading along in the text, you've noticed by now that our definition of measurability is different from your text's. Never-the-less, we will prove all the same (important) theorems. You are welcome to use on this assignment Theorems 16.21 and 16.23, which we will be proving in class shortly.

1. Carothers 16.40
2. Carothers 16.44
3. Carothers 16.45
4. Carothers 16.48
5. Carothers 16.53
6. Carothers 16.58
7. Carothers 16.60
8. Suppose $E \subseteq \mathbb{R}$. Prove that E is measurable if and only if for any $\epsilon > 0$ there is an open set G and a closed set F such that $F \subseteq E \subseteq G$ and $m^*(G \setminus F) < \epsilon$. (This is your text's definition of measurability.)
9. Use the Caratheodory Condition (property CC) to prove that a set $E \subseteq \mathbb{R}$ is measurable if and only if $E \cap (-n, n)$ is measurable for each $n \in \mathbb{N}$.
10. Revisit 16.28 using the full power of the theorems we've developed for Lebesgue measure. That is, try to come up with a tidy short proof that $m(\Delta_\alpha) = \alpha$.