1. A rectangle is a four-sided figure such that the four interior angles are all right angles.

a) Prove that the opposite sides of a rectangle have equal length. Hint: You can use ASA (I-26).

b) Prove that the diagonals have equal length.

c) Prove that the diagonals bisect each other.

Solution:
Let $ABCD$ be a rectangle. By I-27, the opposite sides of the rectangle are parallel. Form the diagonal AC. By I-29, $\angle BAC$ equals $\angle DCA$. Similarly, by I-29, $\angle BCA$ equals $\angle DAC$. By ASA with the common side AC we conclude that $\triangle BAC$ is congruent to $\triangle DCA$. In particular, $|DC| = |AB|$ and $|AD| = |BC|$.

Note that triangle $\triangle BAD$ is congruent to $\triangle CDA$ by SAS: $|AB| = |CD|$, $|AD| = |AD|$, and $\angle BAD$ and $\angle CDA$ are right. Hence the two diagonals AC and BD are equal. Also, $\angle ABD$ equals $\angle DCA$.

Let E be the intersection of the two diagonals. Then by I-29, $\angle BDC$ equals $\angle ADB$ (which equals $\angle DCA$). Since $\triangle EDC$ has two equal angles, it is isosceles and $|ED| = |EC|$. Similar arguments show that $|EB| = |EA|$, and $|EB| = |EC|$. Hence the diagonals bisect each other.

2. Prove that the exterior angles of a regular pentagon add to four right angles.

Solution:
Pick a point in the interior of the pentagon and join it to each vertex. The sum of the interior angles of the resulting triangles is 10 right angles (two for each triangle). Hence the sum of the interior angles of the pentagon is 6 right angles (we get 4 right angles at the vertex in the interior of the pentagon). The sum of the interior angles plus exterior angles is 10 right angles (2 right angles for each vertex). Hence the sum of the exterior angles is 4 right angles.