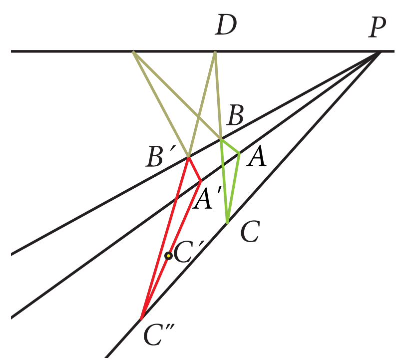
1. Sillwell 6.2.1
Formulate an appropriate statement of Little Desargues if one has parallels.

Solution:
If the lines joining corresponding vertices of two triangles are parallel, and if two of the corresponding sides of the triangles are parallel, then so is the third.

2. Stillwell 6.3.2
Formulate a converse of the Little Desargues theorem and show that it follows from the Little Desargues Theorem.

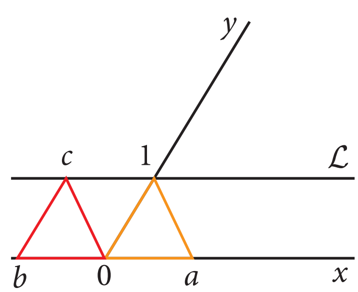
Solution:
The converse is:

Let ABC and $A'B'C'$ be two triangles that are in perspective from a line $\mathcal{L}$. If AA' and BB' intersect at a point P on $\mathcal{L}$, then so does CC'.



To prove it, we proceed as follows. Let C'' be the point of intersection of CP and $A'C'$. Then ABC and $A'B'C''$ are in perspective from P. Now AB and $A'B'$ intersect at a point on $\mathcal{L}$. Also $A'C''$ is the same line as $A'C'$ and hence $A'C''$ and AC also intersect at a point on $\mathcal{L}$. By the Little Desargues theorem, the triangles must be in perspective from $\mathcal{L}$. Let D be the point on $\mathcal{L}$ that is the intersection with BC (and therefore also $B'C''$). Then C'' is on $A'C'$ and on DB'. But C' is on $A'C'$. And $C'B'$ intersects CB on $\mathcal{L}$ at the point D (since ABC and $A'B'C'$ are in perspective from $\mathcal{L}$ by hypotheses). Hence C' is on $A'C'$ and on DB' and is therefore equal to C''. So the triangles are in perspective from P.

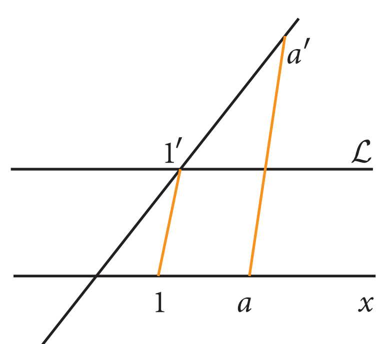
3. Stillwell 6.4.2
Show that each element a on the line has a corresponding element b such that $a + b = 0$.



Solution:
Let 1 be the point of intersection of the y axis and the line $\mathcal{L}$. Consider the triangle $01a$. Form a line through 0 parallel to $1a$. Let c be the point of intersection with $\mathcal{L}$. Now form a line through c parallel to 01. Let b be the intersection of this line with the x axis.

Reversing our steps, it is clear that $a + b = 0$.

4. Stillwell 6.4.3
Show that $1a = a$ for any a.

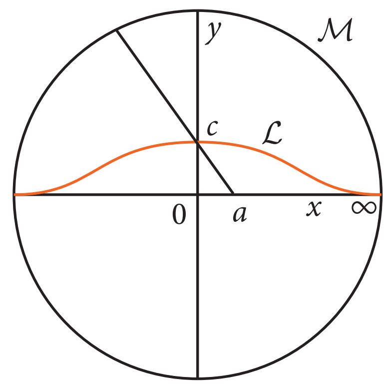


Solution:
Let $1'$ be the intersection of $\mathcal{L}$ with the y axis. To mul-

tiply a by 1 we construct the line through a parallel to $11'$. Let a' be the the intersection of this line with the y axis. We then construct the line through a' that is parallel to $11'$. This is again aa' and its intersection with the x-axis is the point $1a$. But this is exactly a.

5. Stillwell 6.4.5
What happens when we try to construct $a + \infty$?

Solution:
We get ∞, if we interpret things properly.



Let $\mathcal{M}$ be the line at infinity. Let a be a finite point on the x-axis, and let c be the intersection of the y-axis with $\mathcal{L}$. To form $a + \infty$ we start at infinity and form the line parallel to $0c$. What can this line be? Lines parallel to $0c$ all meet at the intersection of the y axis and the line at infinity. In our case, this is exactly the line at infinity. We then form the intersection of this line with $\mathcal{L}$, which is just the point ∞. Finally, we construct the line through this point parallel to ac. Again, this must be the line at infinity, and when we intersect it with the x-axis, we obtain ∞.

6. Stillwell 6.6.1
Let q be a quaterion

$$\begin{pmatrix} a+ib & c+id \\ c-id & a-ib \end{pmatrix}.$$

Prove that q has nonzero determinant if $q \neq 0$.

Solution:
We have

$$\det(q) = (a-ib)(a+ib) - (c+id)(-c+id) = a^2 + b^2 + c^2 + d^2$$

which is nonzero if some entry in q is non-zero.

7. Stillwell 6.6.2
Find two quaterions s and t such that $st \neq ts$.

Solution:
Let $s = i$ and $t = j$ as defined in problem 6.6.3. Then

$$ij = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix} = k,$$

but

$$ji = -k.$$