**1.** Each of the following expressions is $f'(a)$ for some function $f(x)$ at some point $x = a$. Determine what $f(x)$ and $a$ are.

a) $\lim_{h\to 0} \frac{(3+h)^2 - 9}{h}$

b) $\lim_{x\to \pi} \frac{\cos(3x)+1}{x-\pi}$

c) $\lim_{h\to 0} \frac{e^2 - e^{2+h}}{h}$

**Solution, part a:**
$f(x) = x^2, a = 3.$

**Solution, part b:**
$f(x) = \cos(3x), a = \pi$

**Solution, part c:**
$f(x) = -e^x, a = 2$

**2.** Find the formula for the tangent line to the curve

$$y = \frac{1}{x}$$

passing through the point $(2, 1/2)$.

**Solution:**
Let $f(x) = 1/x$. We need to compute $f'(a)$. Now,

$$f'(2) = \lim_{x\to 2} \frac{\frac{1}{x} - \frac{1}{2}}{x-2} = \lim_{x\to 2} \frac{\frac{2-x}{2x}}{x-2} = \lim_{x\to 2} \frac{-1}{2x} = \frac{-1}{4}.$$

Hence the tangent line (which passes through $(2, 1/2)$) is given by

$$y - \frac{1}{2} = -\frac{1}{4}(x-2).$$

**3.** A population of bacteria has size 10000 at time $t = 0$ and doubles every 5 hours.

a) Write a formula for the function $P(t)$ giving bacteria population at time $t$ hours.

b) Determine the rate of growth of the population at time $t = 1$. You will need to know that

$$\lim_{h\to 0} \frac{2^h - 1}{h} = \ln(2).$$

**Solution, part a:**
$P(t) = 10000 \cdot 2^{t/5}$.

**Solution, part b:**
The rate of growth at time $t = 1$ is given by

$$\begin{aligned} P'(1) &= \lim_{h \to 0} \frac{P(1+h) - P(1)}{h} \\ &= \lim_{h \to 0} \frac{10000 \cdot 2^{(1+h)/5} - 10000 \cdot 2^{1/5}}{h} \\ &= 10000 \cdot 2^{1/5} \lim_{h \to 0} \frac{2^{h/5} - 1}{h}. \end{aligned}$$

Now make a substitution, $u = h/5$. We then get

$$P'(1) = 10000 \cdot 2^{1/5} \lim_{u \to 0} \frac{2^u - 1}{5u} = 2000 \cdot 2^{1/5} \lim_{u \to 0} \frac{2^u - 1}{u} = 2000 \cdot 2^{1/5} \ln(2).$$