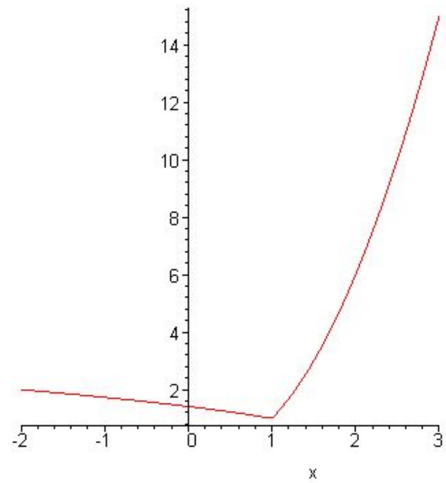
Let

$$f(x) = \begin{cases} \sqrt{2-x} & \text{for } x < 1 \\ 2x^2 - x & \text{for } x \geq 1 \end{cases}$$

1. Sketch the graph of $f(x)$.

Solution:



2. Compute $\lim_{x\to 1^-} f(x)$.

Solution:
$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} \sqrt{2-x} = 1.$

3. Compute $\lim_{x\to 1^+} f(x)$.

Solution:
$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} 2x^2 - x = 1.$

4. Does $\lim_{x\to 1} f(x)$ exist? If so, what is the limit equal to?

Solution:
Yes, and $\lim_{x\to 1} f(x) = 1.$

5. Is $f(x)$ continuous at $x = 1$?

Solution:
Yes. We see $\lim_{x\to 1} f(x) = f(1) = 1$, so f is continuous at $x = 1$.

For the problems below, you may use the fact that $\lim_{x\to 0} \frac{\sin(x)}{x} = 1$.

1. Compute $\lim_{x\to 0} \frac{\tan(x)}{x}$.

Solution:

$$\begin{aligned}\lim_{x\to 0}\frac{\tan(x)}{x} &= \lim_{x\to 0}\frac{\sin(x)}{x\cos(x)}\\ &= \lim_{x\to 0}\left[\frac{1}{\cos(x)}\cdot\frac{\sin(x)}{x}\right]\\ &= \lim_{x\to 0}\frac{1}{\cos(x)}\cdot\frac{\sin(x)}{x}\\ &= 1.\end{aligned}$$

2. Compute $\lim_{x\to 0}\frac{[\sin(x)]^6}{8x^6}$.

Solution:

$$\begin{aligned}\lim_{x\to 0}\frac{[\sin(x)]^6}{8x^6} &= \frac{1}{8}\lim_{x\to 0}\left(\frac{\sin(x)}{x}\right)^6\\ &= \frac{1}{8}\left(\lim_{x\to 0}\frac{\sin(x)}{x}\right)^6\\ &= \frac{1}{8}(1^6)\\ &= \frac{1}{8}.\end{aligned}$$

3. Compute $\lim_{x\to 0^+}\frac{\sin(x)}{\sqrt{x}}$.

Solution:

$$\begin{aligned}\lim_{x\to 0^+}\frac{\sin(x)}{\sqrt{x}} &= \lim_{x\to 0^+}\frac{\sin(x)}{\sqrt{x}}\cdot\frac{\sqrt{x}}{\sqrt{x}}\\ &= \lim_{x\to 0^+}\frac{\sin(x)}{x}\cdot\lim_{x\to 0^+}\sqrt{x}\\ &= 1\cdot 0\\ &= 0.\end{aligned}$$

4. Compute $\lim_{x\to 0}\frac{\sin(7x)}{x}$.

Solution:

$$\lim_{x\to 0}\frac{\sin(7x)}{x} = 7\cdot\lim_{x\to 0}\frac{\sin(7x)}{7x}.$$

Now, let $u = 7x$. We can see that as $x \to 0$, $u \to 0$ as well. Therefore,

$$\begin{aligned}\lim_{x\to 0}\frac{\sin(7x)}{7x} &= \lim_{u\to 0}\frac{\sin(u)}{u}\\ &= 1.\end{aligned}$$

Thus,

$$\lim_{x \to 0} \frac{\sin(7x)}{x} = 7.$$

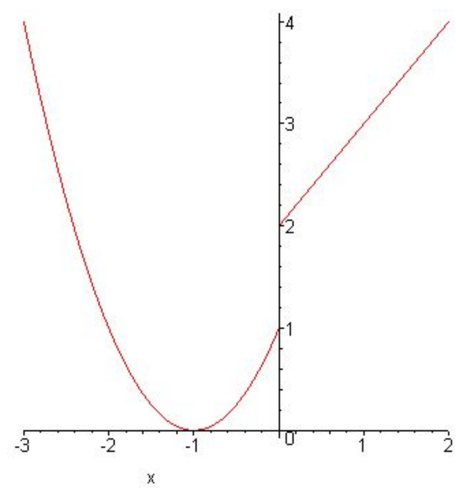
Consider the function

$$g(x) = \begin{cases} [a(x+1)]^2 & x \leq 0 \\ x + b & x \geq 0 \end{cases}$$

where a and b are constants.

1. Sketch the graph of $g(x)$ when $a = 1$ and $b = 2$.

Solution:



2. Determine all values of a and b such that $g(x)$ is continuous at $x = 0$.

Solution:
We see that $g(x)$ will be continuous if $[a(x + 1)]^2 = x + b$, at $x = 0$. In other words, where $a^2 = b$.