1. Compute the following derivative:

$$\frac{d}{dx}\int_0^{\sqrt{x}} \cos(t)\ dt.$$

Solution:

$$\frac{d}{dx}\int_0^{\sqrt{x}} \cos(t)\ dt = \cos(\sqrt{x}) \cdot \frac{d}{dx}(\sqrt{x}) = \frac{\cos(\sqrt{x})}{2\sqrt{x}}.$$

2. Water is filling a cylindrical tank at a rate of e^{-2t} meters^3/second. The tank has radius 2 meters. What is the change in height of the water from time $t = 1$ to time $t = 2$?

Solution:
Since the tank is filling at a rate of e^{-2t} m^3/s, then the change in volume between $t = 1$ and $t = 2$ is given by:

$$\begin{aligned} V &= \int_1^2 e^{-2t}\ dt \\ &= -\frac{e^{-2t}}{2}\bigg|_1^2 \\ &= -\frac{e^{-4}}{2} + \frac{e^{-2}}{2} \approx .0585 \text{ m}^3. \end{aligned}$$

So between $t = 1$ and $t = 2$ the volume has increased by about $.0585$ cubic meters. To find the change in height, we plus this into the equation for the volume of a cylinder and get

$$h = \frac{V}{\pi r^2} \approx \frac{.0585}{4\pi} \approx .00466 \text{ m}^3.$$

3. Compute the following antiderivatives.

a) $\displaystyle\int \cos(x)e^{\sin(x)}\ dx$

b) $\displaystyle\int \frac{dx}{x\ln(x)}$

c) $\displaystyle\int x\sin(2x^2)\ dx$

Solution:

a) Letting $u = \sin(x)$ we get $du = \cos(x)\ dx$ and thus

$$\int \cos(x)e^{\sin(x)}\ dx = \int e^u\ du = e^u + C = e^{\sin(x)} + C.$$

b) Letting $u = \ln(x)$ we get $du = \frac{dx}{x}$ and thus

$$\int \frac{dx}{x\ln(x)} = \int \frac{du}{u} = \ln(u) + C = \ln(\ln(x)) + C.$$

c) Letting $u = 2x^2$, we get $du = 4x\ dx$ and thus

$$\int x\sin(2x^2)\ dx = \int \frac{1}{4}\sin(u)\ du = -\frac{\cos(u)}{4} + C = -\frac{\cos(2x^2)}{4} + C.$$