

Name: _____

Id: _____

Rules:

You have 60 minutes to complete the exam.

Partial credit will be awarded, but you must show your work.

All derivatives must be computed directly from the definition using limits.

No calculators, books, notes, or other aids are permitted.

Turn off anything that might go beep during the exam.

If you need extra space, you can use the back sides of the pages. Please make it obvious when you have done so.

Good luck!

Problem	Possible	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

1. (20 points)

Let

$$f(x) = \frac{e^x + 1}{e^x - 1}.$$

- a What is the domain of $f(x)$?
- b Find a formula for the inverse function $f^{-1}(x)$.
- c What is the domain of the inverse function $f^{-1}(x)$?

2. (20 points)

Compute the following limits. You must justify your steps to receive full credit.

a $\lim_{x \rightarrow 1^+} \frac{x - 2}{1 - x}$

b $\lim_{x \rightarrow \infty} \frac{2x(x - \sqrt{x})}{4 - 5x^2}$

c $\lim_{x \rightarrow -2} \frac{x^2 - 4}{(|x| - 2)}$

3. (20 points)

Let

$$f(x) = \begin{cases} 4x - 12 & x < 3 \\ a & x = 3 \\ \sin(x + b) & x > 3 \end{cases}$$

where a and b are constants.**a** Compute $\lim_{x \rightarrow 3^-} f(x)$. Justify your work.**b** Compute $\lim_{x \rightarrow 3^+} f(x)$. Justify your work.**c** Find all values of a and b such that $f(x)$ is continuous at $x = 3$.

4. (20 points)

A car is travelling on a straight section of the Parks highway at night. At time $t = 0$ the driver spots a moose ahead on the road and starts to slow the vehicle. The position of the car during the time interval $0 \leq t \leq 10$ is given by

$$p(t) = \frac{1}{12}(t^3 - 30t^2 + 300t)$$

where $p(t)$ is given in meters and t is measured in seconds. Recall that for the following problems, any derivatives must be computed directly from the definition using limits.

a Find the average velocity of the car during the time interval $0 \leq t \leq 5$.

b Find the instantaneous velocity of the car at the time $t = 5$.

5. (20 points)

Find the formula for the tangent line to the curve

$$y = \frac{1}{x}$$

passing through the point $(2, \frac{1}{2})$. Any derivatives must be computed directly from the definition using limits.