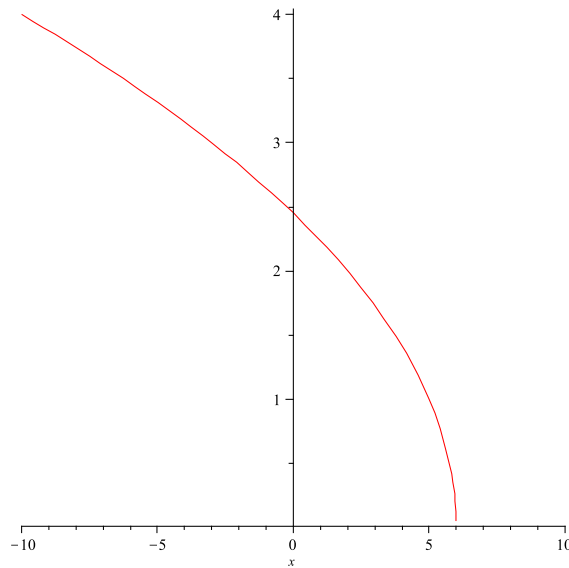


CALCULUS HOMEWORK SOLUTIONS
WEEK 5

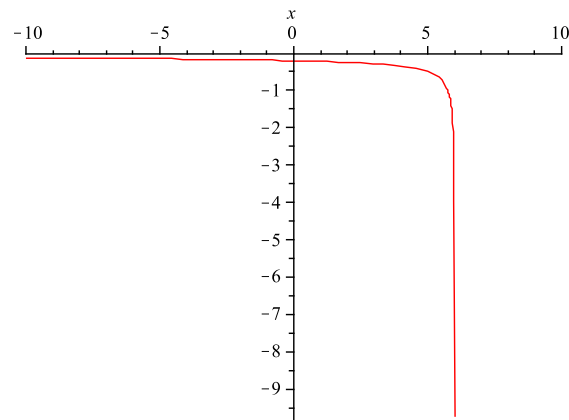
§2.8, 30a: Sketch the graph of $f(x) = \sqrt{6-x}$ by starting with the graph of $y = \sqrt{x}$ and using the transformations of Section 1.3.

Solution: If $y(x) = \sqrt{x}$, then we can see that $f(x) = y(-x+6)$ will be achieved by flipping the graph of \sqrt{x} about the y -axis, and then shifting the graph to the right:



§2.8, 30b: Use the graph from part (a) to sketch the graph of f' .

Solution: From the above graph, we can see that as x goes to 6 from the right, the slope of the graph goes to negative infinity. Also, as x goes to negative infinity, we see that the slope of the graph remains negative, but it does approach zero. So the graph of f' will look like this:



§2.8, 30c: Use the definition of a derivative to find $f'(x)$. What are the domains of f and f' .

Solution:

$$\begin{aligned}
 f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\sqrt{6-x-h} - \sqrt{6-x}}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\sqrt{6-x-h} - \sqrt{6-x}}{h} \cdot \frac{\sqrt{6-x-h} + \sqrt{6-x}}{\sqrt{6-x-h} + \sqrt{6-x}} \\
 &= \lim_{h \rightarrow 0} \frac{6-x-h-6+x}{h(\sqrt{6-x-h} + \sqrt{6-x})} \\
 &= \lim_{h \rightarrow 0} \frac{-h}{h(\sqrt{6-x-h} + \sqrt{6-x})} \\
 &= \lim_{h \rightarrow 0} \frac{-1}{\sqrt{6-x-h} + \sqrt{6-x}} \\
 &= -\frac{1}{2\sqrt{6-x}}.
 \end{aligned}$$

The domain of f is anywhere $6-x \geq 0$, or where $x \leq 6$. The domain of f' is anywhere $6-x \geq 0$, and anywhere $6-x \neq 0$. So the domain of f' is $x < 6$.

§2.8, 43:

Solution: See solution in the back of the book.

§2.8, 49:

Solution: See solution in the back of the book.

§2.8, 51:

Solution: See solution in the back of the book.

§3.1, 46: Find the first and second derivatives of the function $G(r) = \sqrt{r} + \sqrt[3]{r}$

Solution: Notice $G(r) = \sqrt{r} + \sqrt[3]{r} = r^{1/2} + r^{1/3}$. From the addition and power rules, we get:

$$\begin{aligned}
 G'(r) &= \frac{r^{-1/2}}{2} - \frac{r^{-2/3}}{3} \\
 &= \frac{1}{2\sqrt{r}} - \frac{1}{3\sqrt[3]{r^2}}.
 \end{aligned}$$

We differentiate $G'(r)$, again using the addition and power rules, which gives us:

$$\begin{aligned} G''(r) &= -\frac{r^{-3/2}}{4} + \frac{2r^{-5/3}}{9} \\ &= -\frac{1}{4\sqrt{r^3}} + \frac{2}{9\sqrt[3]{r^5}}. \end{aligned}$$

§3.1, 54: Find an equation of a tangent line to the curve $y = x\sqrt{x}$ parallel to the line $y = 1 + 3x$.

Solution: Let $f(x) = x\sqrt{x}$, and let $g(x) = 1 + 3x$. We first need to find the point x_0 such that

$$(1) \quad g'(x) = f'(x_0).$$

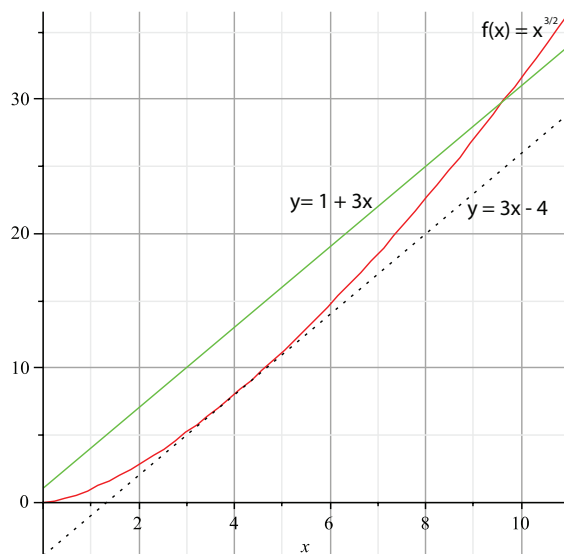
Notice that $g'(x) = 3$. Also notice that $f(x)$ can be written as $f(x) = x^{3/2}$, and so

$$f'(x) = \frac{3}{2}x^{1/2}.$$

Plugging this into equation (1) gives us:

$$3 = \frac{3\sqrt{x_0}}{2}$$

From this, we get that $\sqrt{x} = 2$, or $x = 4$. So we know the tangent line to f is parallel to g at the point $(4, 8)$. The equation of the tangent line to f going through the point $(4, 8)$ is $y = 3x - 4$.



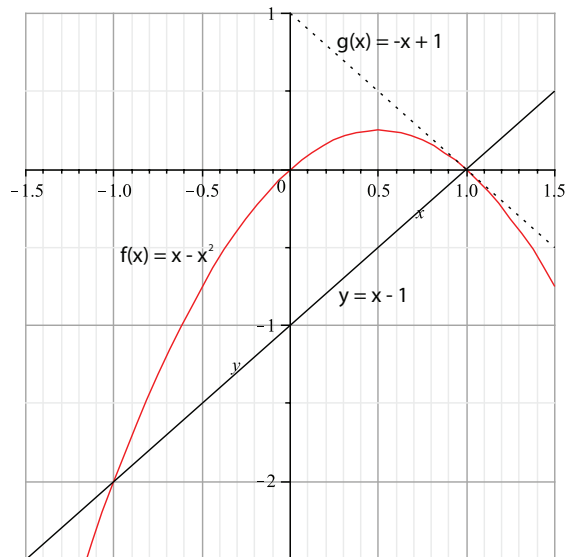
§3.1, 58: Where does the normal line to the parabola $y = x - x^2$ at the point $(1, 0)$ intersect the parabola a second time? Illustrate with a sketch.

Solution: Let $f(x) = x - x^2$. So $f'(x) = 1 - 2x$, and thus, $f'(1) = -1$. Notice that the equation of the line tangent to $f(1)$ is $g(x) = -x + 1$. The normal line to $f(1)$ will have a slope of 1. The equation of the line with a slope of 1 and going through the point $(1, 0)$ is $y = x - 1$. To

find out where y intersects f , we set $y - f = 0$, giving us:

$$\begin{aligned} 0 &= x - 1 - x + x^2 \\ &= x^2 - 1 \end{aligned}$$

So y intersects f at $x = 1$ and $x = -1$, or at the points $(1, 0)$ and $(-1, -2)$. So the normal line to f at the point $(1, 0)$ intersects the parabola a second time at the point $(-1, -2)$.



§3.1, 61: Use the definition of the derivative to show that if $f(x) = 1/x$, then $f'(x) = -1/x^2$.

Solution: From the definition, we have

$$\begin{aligned} \lim_{h \rightarrow 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} &= \lim_{h \rightarrow 0} \frac{\frac{x}{x(x+h)} - \frac{x+h}{x(x+h)}}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{-h}{x(x+h)}}{h} \\ &= \lim_{h \rightarrow 0} \frac{-h}{x(x+h)} \cdot \frac{1}{h} \\ &= \lim_{h \rightarrow 0} \frac{-1}{x(x+h)} \\ &= \frac{1}{x^2}. \end{aligned}$$

§3.1, 78: Draw a diagram showing two perpendicular lines that intersect on the y -axis and are both tangent to the parabola $y = x^2$.

Solution: Let $f(x) = x^2$. We can see that $f'(x) = 2x$. We need to find two points x_1, x_2 such that $f'(x_1) = -1/f'(x_2)$. Also, since f is an even function, we know $x_2 = -x_1$. These two equations give us:

$$2x_1 = \frac{1}{2x_1}.$$

From this, we get

$$4x_1^2 - 1 = 0.$$

Thus, $x_1 = \frac{1}{2}$, and $x_2 = -\frac{1}{2}$. To find where the two points meet, we find the equation of the line tangent to $(x_1, f(x_1))$, or the point $(1/2, 1/4)$. Plugging this into the point-slope formula gives us:

$$y = x - 1/4.$$

So this line crosses the y -axis at $y = -1/4$. By a similar computation, we can see that the tangent line to $f(x_2)$ also crosses the y -axis at $y = -1/4$.

