CALCULUS HOMEWORK SOLUTIONS
WEEK 11

§4.8, 8: Use Newton's method with the initial approximation $x_1 = -1$ to find x_3, the third approximation of the root of the equation: $f(x) = x^5 + 2$. (Give your answer to four decimal places.)

Solution: Notice $f'(x) = 5x^4$. Applying Newton's method, we get:

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = -1 - \frac{(-1)^5 + 2}{5(-1)^4} = -1.2$$

Applying Newton's method again, we get:

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = -1.2 - \frac{(-1.2)^5 + 2}{5(-1.2)^4} \approx -1.1529.$$

§4.8, 18: Use Newton's method to find the roots of the equation $e^x = 3 - 2x$ correct to six decimal places.

Solution: Based on the graphs of e^x and $3 - 2x$, we know the equation has only one root. Let $f(x) = 3 - 2x - e^x$, and let $x_1 = .5$ Applying Newton's method, we get:

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = .5 - \frac{3 - 2(.5) - e^{.5}}{-2 - e^{.5}} \approx .596274$$

Repeating this process, we get:

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = .596274 - \frac{3 - 2(.596274) - e^{.596274}}{-2 - e^{.596274}} \approx .594206$$

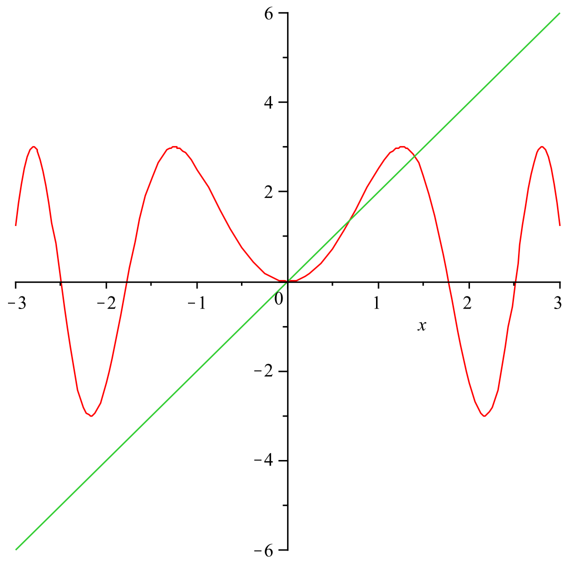
$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = .594206 - \frac{3 - 2(.594206) - e^{.594206}}{-2 - e^{.594206}} \approx= .594205$$

$$x_5 = x_4 - \frac{f(x_4)}{f'(x_4)} = .594205 - \frac{3 - 2(.594205) - e^{.594205}}{-2 - e^{.594205}} \approx .594205.$$

Since $x_4 \approx x_5$, we can stop here. So we have $e^x = 3 - 2x$ at $x \approx 0.594205$.

§4.8, 26: Use Newton's method to find all roots of the equation $3\sin(x^2) = 2x$ correct to eight decimal places. Start by drawing a graph to find initial approximations.

Solution: Below is the graph of $y = 3\sin(x^2)$ (in red) and $y = 2x$. (in green):



From this graph, we can see that $x = 0$ is a root of the equation, and that there are two others. Let $f(x) = 3\sin(x^2) - 2x$. To find the first root, we let $x_1 = .5$. From Newton's method, we get:

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = .5 - \frac{3\sin((.5)^2) - 2(.5)}{3\cos((.5)^2)\cdot 2(.5) - 2} \approx .78430300.$$

Repeating this process gives us:

$$x_3 \approx .69609321$$
$$x_4 \approx .69300735$$
$$x_5 \approx .69299996$$
$$x_6 \approx .69299996$$

So we have a root at $x \approx .69299996$. To find the other root, we repeat this, letting $x_1 = 1.5$.

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1.5 - \frac{3\sin((1.5)^2) - 2(1.5)}{3\cos((1.5)^2)\cdot 2(1.5) - 2} \approx 1.41301039$$
$$x_3 \approx 1.39594393$$
$$x_4 \approx 1.39525190$$
$$x_5 \approx 1.39525077$$
$$x_6 \approx 1.39525077$$

So the other root is at $x \approx 1.39525077$. So we have $3\sin(x^2) = 2x$ at $x = 0$ and $x \approx 0.69299996$ and $x \approx 1.39525077$.

§4.9, 6: Find the most general antiderivative of the function $f(x) = x(2-x)^2$. (Check your answer by differentiation)

Solution: Notice

$$f(x) = x(2-x)^2 = x(4 - 4x + x^2) = 4x - 4x^2 + x^3.$$

So the power rule, we get the general derivative $F(x) = 2x^2 - \frac{4x^3}{3} + \frac{x^4}{4} + C$. If we differentiate, we can see that $F'(x) = f(x)$.

§4.9, 20: Find the most general antiderivative of the function

$$f(x) = \frac{2+x^2}{1+x^2}.$$

Check your answer by differentiation.

Solution: Notice

$$f(x) = \frac{2+x^2}{1+x^2} = \frac{(1+x^2)+1}{1+x^2} = \frac{1+x^2}{1+x^2} + \frac{1}{1+x^2} = 1 + \frac{1}{1+x^2}.$$

Thus, we get the general antiderivative $F(x) = x + \tan^{-1}(x) + C$. Differentiating this gives us $F'(x) = f(x)$.

§4.9, 44: Find f if $f''(t) = 2e^t + 3\sin(t)$, $f(0) = 0$, $f(\pi) = 0$.

Solution: We first find the general antiderivative of $f''(t)$:

$$f'(t) = 2e^t - 3\cos(t) + C.$$

We next find the general antiderivative of $f'(t)$:

$$f(t) = 2e^t - 3\sin(t) + Ct + D.$$

Since $f(0) = 0$, we see

$$f(0) = 2e^0 - 3\sin(0) + C(0) + D = 0.$$

So $2 + D = 0$, thus $D = -2$. Since $f(\pi) = 0$, we have:

$$f(\pi) = 2e^\pi - 3\sin(\pi) + C(\pi) - 2 = 0.$$

Thus, $C = \frac{2-2e^\pi}{\pi}$. This gives us:

$$f(t) = 2e^t - 3\sin(t) + \left(\frac{2-2e^\pi}{\pi}\right) t - 2.$$