Quiz 7
Math 314

Name: Key
4/11/08

1. Give an equilibrium vector for the stochastic matrix

$$A = \begin{pmatrix} .6 & .2 \\ .4 & .8 \end{pmatrix}.$$

Make sure the entries of your answer add up to 1.

$$A - I = \begin{pmatrix} -.4 & .2 \\ .4 & -.2 \end{pmatrix} \longrightarrow \begin{pmatrix} -.4 & .2 \\ 0 & 0 \end{pmatrix} \to \begin{pmatrix} -2 & 1 \\ 0 & 0 \end{pmatrix}$$

So $\begin{matrix} -2x + y = 0 \\ y \text{ free} \end{matrix} \Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{1}{2}y \\ y \end{pmatrix} = y\begin{pmatrix} \frac{1}{2} \\ 1 \end{pmatrix}$ so $\begin{pmatrix} \frac{1}{2} \\ 1 \end{pmatrix}$ is a steady state vector. To make the entries add to one

$$\frac{1}{1+\frac{1}{2}}\begin{pmatrix} \frac{1}{2} \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{3} \\ \frac{2}{3} \end{pmatrix}$$

2. Let $B = \begin{pmatrix} 3 & -1 & 0 \\ 0 & 6 & 1 \\ 0 & 0 & 2 \end{pmatrix}$

(a) What are the eigenvalues of B?

3, 6, 2

(b) Find an eigenvector for the largest of the eigenvalues.

$\lambda = 6$

$$A - \lambda I = \begin{pmatrix} -3 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -4 \end{pmatrix} \longrightarrow \begin{pmatrix} -3 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \qquad \begin{matrix} -3x - y = 0 \\ z = 0 \\ y \text{ free} \end{matrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -\frac{1}{3}y \\ y \\ 0 \end{pmatrix} = y\begin{pmatrix} -\frac{1}{3} \\ 1 \\ 0 \end{pmatrix} \text{ so } \begin{pmatrix} -\frac{1}{3} \\ 1 \\ 0 \end{pmatrix} \text{ or } \begin{pmatrix} -1 \\ 3 \\ 0 \end{pmatrix}$$

is an eigenvector with eigenvalue 6.