
Math 200 Calculus I
Final Exam
19 December 2002

Name: _____

This exam contains 10 questions worth a total of 100 points. You may not use a calculator or notes of any kind. Show your work to receive full credit.

1. (10 points) Evaluate the following limits. If the limit is infinite (positive or negative), state this. (Don't just write that it does not exist.)

(a) $\lim_{x \rightarrow 2} \frac{x^3 - 4x}{x^2 - x - 2}$

(b) $\lim_{x \rightarrow -3^+} \frac{x^2}{x + 3}$

2. (10 points) Find the derivative for the functions below. Simplify your answers.

(a) $f(x) = x^2 \csc(2 - 3x)$

(b) $f(x) = \frac{x^2}{x+3}$ [Use the quotient rule.]

3. (15 points) Evaluate the following integrals.

(a) $\int 3x \sin\left(1 - \frac{x^2}{3}\right) dx$

(b) $\int_1^4 \frac{3t^2 - 1}{\sqrt{t}} dt$

(c) $\int_{\pi/8}^0 \frac{\tan^2(2x) \sec^2(2x)}{4} dx$

4. (10 points) Find $\frac{da}{dc}$ for $a^2 - \sec c + 3ac = 0$.

5. (10 points) Assume the velocity of a body moving along a coordinate line is given by $v(t) = \sin t$ and its initial position is 2. (i.e. $s(0) = 2$) Find the body's position at time t .

6. (5 points) Write the equation of the line tangent to the graph of $f(x) = x^{2/3}$ at $x = 8$.

7. (10 points) Given $f(x) = 3x(x + 2)^{-1}$, $f'(x) = 6(x + 2)^{-2}$, and $f''(x) = -12(x + 2)^{-3}$, answer the questions below.

(a) Find any horizontal asymptotes or state that none exist. Explain your answer.

(b) Find any vertical asymptotes or state that none exist. Explain your answer.

(c) Find the intervals on which $f(x)$ is concave up and concave down. Explain your answer.

(d) Find any points of inflection or state that none exist.

8. (10 points) An open topped rectangular box with a square base is to have a surface area of 48 square feet. What dimensions will maximize the volume of the box?

9. (10 points) The region R bounded by $y = \sqrt{x}$ and $y = x/2$ is revolved about the axis $y = 3$. [I recommend you sketch and label the region and axis of rotation below.]

(a) Find the volume of the solid of revolution using the washer method. (Set up the integral only.)

(b) Find the volume of the solid of revolution using cylindrical shells. (Set up the integral only.)

10. (10 points) How much work does it take to pump the water from a full upright circular cylindrical tank of radius 3 feet and height 5 feet to a level 4 feet above the top of the tank. (Assume water weighs 62.4 pounds per cubic foot.) Set up the integral only. Remember $W = F * d$.