

Name _____

There are seven questions worth a total of 100 points. You must show your work to receive credit for your answer.

1. (5 points each) Evaluate the following limits. If the limit is infinite, please state whether it is $+\infty$ or $-\infty$. Please indicate an application of L'Hopital's rule with an "H."

(a) $\lim_{x \rightarrow \infty} \frac{(3x+1)(2x-5)}{4x^2-20}$

(b) $\lim_{x \rightarrow 5^-} \frac{(x-4)^2}{x^2-25}$

(c) $\lim_{x \rightarrow \infty} 8x \ln\left(1 + \frac{2}{x}\right)$

2. (5 points each) Find y' for each of the following expressions.

(a) $y = [x^2 - 3 \cos(4x)]^{1/3}$

(b) $y = \sin(5xe^{2-x})$

(c) $\ln y = \frac{3}{x} - xy$

(d) $y = \int_{10}^{\sqrt{x}} (\ln(t))^{4/5} dt$

3. (5 points each) Evaluate the following indefinite integrals.

(a) $\int \frac{1 + 2\sqrt{x} - 3x^5}{x} dx$

(b) $\int \frac{1}{2} x e^{3x^2} dx$

(c) $\int \frac{\sec^2(\sqrt{x})}{\sqrt{x}} dx$

(d) $\int \frac{dy}{y \ln y}$

4. (5 points each) Evaluate the following definite integrals.

(a) $\int_3^0 \sqrt{9 - r^2} \, dr$

(b) $\int_0^{7/2} (2y + 1)^{1/3} \, dy$

5. (6 points each) Find the absolute maximum and the absolute minimum of f (if any) on the given interval. You must SHOW YOUR WORK. If there is no absolute max/min, you must state this explicitly.

(a) $f(x) = x^{2/3}(20 - x)$ on $[-1, 20]$

(b) $f(x) = \frac{x^2}{x+1}$ on $(-5, -1)$

6. (10 points) Sketch the graph of a function f with the following properties. Your graph should be large and neat and accurate!

(a) f is defined for all real numbers.

(b) $f(0) = 2$ and $f(1) = 0$

(c) $f' \geq 0$ on $(1, \infty)$ and $f' \leq 0$ on $(-\infty, 1)$

(d) $f'(1)$ is undefined and $f'(-3) = 0$

(e) $f'' > 0$ on $(-\infty, -3) \cup (1, +\infty)$ and $f'' < 0$ on $(-3, 1)$

7. Suppose the acceleration function of a particle moving along a straight line is $a(t) = 4 \text{ m/s}^2$ and that the position and velocity at time $t = 1 \text{ s}$ are $s = 20 \text{ m}$ and $v = -8 \text{ m/s}$.

(a) (6 points) Find the equations of $s(t)$ and $v(t)$.

(b) (2 points) At $t = 1$ is the particle speeding up or slowing down? Explain your answer.

(c) (5 points) Assuming the only information you have about the particle is its velocity, determine the distance traveled by the particle from $t = 0$ to $t = 3$.