

MATH 200X
Calculus I
Fall 2006

Exam II

Monday October 30, 2006

NAME: _____

This exam contains 7 questions worth a total of 100 points. You may not use books, notes, or calculators for this exam. Show your work to receive full credit. Your work must be organized, clear, and correct. Please circle, or otherwise make clear, your answer to each problem.

Note that a formula sheet is at the back of the exam.

1. (10 points each) Find the derivative for each of the following. You do not have to simplify unless explicitly asked.

(a) $f(x) = \frac{3-2x}{4x+1}$ (Use the quotient rule. Simplify your answer.)

(b) $g(x) = e^{2x} \cos^{-1}(x^2) + \sqrt{e}$

(c) $y = \sqrt{x + \tan(3x)}$

2. (10 points) Determine the absolute maximum and absolute minimum of $f(x) = (x^2 - 1)^3$ on $[-1, 2]$.

3. (10 points) A particle moves along a horizontal line so that its position, s , at time t is $s = t + \sin(2t)$ for $t \geq 0$.

(a) Find the velocity and acceleration functions.

(b) Does the particle change direction at any time between $t = 0$ and $t = \pi/2$? Explain your answer in complete sentences.

4. (10 points) Use linearization or differentials to approximate $\sqrt[3]{1001}$.

5. (10 points) Use logarithmic differentiation to find dy/dx for $y = (x + 1)^{\sin x}$.
Express your answer in terms of x .

6. (10 points) A vertical cylindrical tank 5 feet tall and with diameter 3 feet is filled with water. How fast is the height of the water changing if the we pump out the water at a rate of 2 cubic feet per minute? (Note that the volume of a cylinder is $V = \pi r^2 h$ where r is the radius of the base and h is the height of the cylinder.)

7. Given $x^2 + xy + y^2 = 1$.

(a) (10 points) Find dy/dx in terms in x and y .

(b) (10 points) Find all points where the tangent line is parallel to the line $y = -x$.

EXTRA CREDIT:

Use the properties of inverse functions and knowledge of the standard trigonometric functions to derive the formula

$$\frac{d}{dx} (\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$$

FORMULAS:

$$\frac{d}{dx} (\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx} (\cos^{-1}(x)) = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx} (\tan^{-1}(x)) = \frac{1}{1-x^2}$$