MATH 200
Exam II Solutions

(1) (a) $f'(x) = \frac{-14}{(4x+1)^2}$
(b) $g'(x) = 2e^{2x}[\cos^{-1}(x^2) - \frac{x}{\sqrt{1+x^4}}]$
(c) $y' = (\frac{1}{2})(x + \tan(3x))^{-1/2}(1 + 3\sec^2(3x))$

(2) $f' = 6x(x^2 - 1)^2$ which is zero at $x = 0, 1$, and -1.

Check critical numbers and endpoints:

x	y
0	-1
1	0
-1	0
2	27

Conclusion: f has a maximum of 27 which occurs at $x = 2$ and f has a minimum of -1 which occurs at $x = 0$.

(3) (a) $v = s' = 1 + 2\cos(2t)$, $a = s'' = -4\sin(2t)$

(b) Yes. The particle does change direction. We know this because the derivative changes sign. Specifically, s' is positive at $t = 0$ and negative at $t = \pi/2$.

(4) $f(x) = x^{1/3}$; $f'(x) = (\frac{1}{3})x^{-2/3}$; $a = 1000$; So $f(a) = 10$ and $f'(a) = 1/300$. So $L(x) = (1/300)(x - 1000) + 10$. So $L(1001) = 10 + 1/300 = 10.00333$.

(5) $\ln(y) = \sin x \ln(x + 1)$; Now we do implicit differentiation:
$(1/y) * y' = \cos x \ln(x + 1) + \sin x * \frac{1}{x+1}$;
Solve for y' to get:
$y' = (x + 1)^{\sin x}[\cos x \ln(x + 1) = \frac{\sin x}{x+1}]$

(6) We want dh/dt. We are given: $dV/dt = -2$; $r = 3/2$ and $h = 5$. Note that as the height of the water falls, the radius does not change. Thus we have the equation:
$V = \pi(3/2)^2 h$ or $V = 9\pi/4h$.
Now we differentiate implicitly:
$dV/dt = (9\pi/4)(dh/dt)$.
Now plug in the given values and solve for dh/dt : So $dh/dt = -8/9\pi$ feet per minute.

(7) (a) Differentiate implicitly: $2x + 1 * y + x + y' + 2yy' = 0$
Solve for y' :
$y' = -\frac{2x_y}{x+2y}$

(b) Set $y' = -1$ and solve for y to get $x = y$. Now plug $x = y$ into the original equation to get: $3x^2 = 1$. So $x = \pm 1/\sqrt{3}$.
Final Answer: $(1/\sqrt{3}, 1/\sqrt{3}), (-1/\sqrt{3}, -1/\sqrt{3})$